Chapter 2

FAO Penman-Monteith equation

This chapter introduces the user to the need to standardize one method to compute reference evapotranspiration (ET_o) from meteorological data. The FAO Penman-Monteith method is recommended as the sole ET_o method for determining reference evapotranspiration. The method, its derivation, the required meteorological data and the corresponding definition of the reference surface are described in this chapter.

From the original Penman-Monteith equation (Equation 3) and the equations of the aerodynamic (Equation 4) and surface resistance (Equation 5), the FAO Penman-Monteith method to estimate ET_o can be derived (Box 6):

$$ET_o = \frac{0.408\,\Delta\,(R_n - G) + \gamma\,\frac{900}{T+273}\,u_2\,(e_s - e_a)}{\Delta + \gamma\,(1 + 0.34\,u_2)} \qquad (6)$$

where

ET_o	reference evapotranspiration [mm day^{-1}],
R_n	net radiation at the crop surface [MJ m^{-2} day^{-1}],
G	soil heat flux density [MJ m^{-2} day^{-1}],
T	mean daily air temperature at 2 m height [°C],
u_2	wind speed at 2 m height [m s^{-1}],
e_s	saturation vapour pressure [kPa],
e_a	actual vapour pressure [kPa],
e_s-e_a	saturation vapour pressure deficit [kPa],
Δ	slope vapour pressure curve [kPa °C^{-1}],
γ	psychrometric constant [kPa °C^{-1}].

$$ETP_{P\text{-}M} = \frac{0.408 \times s \ (Rn - G) + \dfrac{\gamma \times 900 \times u_2 \ (e_s - e_a)}{T + 273}}{s + \gamma \ (1 + 0.34 \ u_2)}$$

em que:

Rn – radiação liquida, em MJ m^{-2} dia^{-1};

G – densidade do fluxo de calor, em MJ m^{-2} dia^{-1};

T – temperatura média diária do ar, em ºC;

γ – constante psicométrica (0.063 KPa $ºC^{-1}$);

s –declividade da curva de saturação de vapor, em KPa $ºC^{-1}$;

U_2 – velocidade média diária do vento a 2 metros de altura, em ms^{-1};

e_a – pressão parcial de vapor, em KPa;

e_s - pressão de saturação de vapor, média diária, em KPa.

Atmospheric pressure (P)

The atmospheric pressure, P, is the pressure exerted by the weight of the earth's atmosphere. Evaporation at high altitudes is promoted due to low atmospheric pressure as expressed in the psychrometric constant. The effect is, however, small and in the calculation procedures, the average value for a location is sufficient. A simplification of the ideal gas law, assuming 20°C for a standard atmosphere, can be employed to calculate P:

$$P = 101.3\left(\frac{293 - 0.0065\,z}{293}\right)^{5.26} \quad (7)$$

where P atmospheric pressure [kPa],
z elevation above sea level [m],

Values for atmospheric pressure as a function of altitude are given in Annex 2 (Table 2.1).

$$\gamma = \frac{c_p \ P}{\varepsilon \ \lambda} = 0.665 \times 10^{-3} \ P \qquad (8)$$

where

	γ	psychrometric constant [kPa °C^{-1}],
	P	atmospheric pressure [kPa],
	λ	latent heat of vaporization, 2.45 [MJ kg^{-1}],
	c_p	specific heat at constant pressure, 1.013 10^{-3} [MJ kg^{-1} °C^{-1}],
	ε	ratio molecular weight of water vapour/dry air = 0.622.

The specific heat at constant pressure is the amount of energy required to increase the temperature of a unit mass of air by one degree at constant pressure. Its value depends on the composition of the air, i.e., on its humidity. For average atmospheric conditions a value c_p = 1.013 10^{-3} MJ kg^{-1} °C^{-1} can be used. As an average atmospheric pressure is used for each location (Equation 7), the psychrometric constant is kept constant for each location. Values for the psychrometric constant as a function of altitude are given in Annex 2 (Table 2.2).

EXAMPLE 2

Determination of atmospheric parameters.

Determine the atmospheric pressure and the psychrometric constant at an elevation of 1 800 m.

With:	z =	1 800	m
From Eq. 7:	$P = 101.3\,[(293 - (0.0065)\,1800)/293]^{5.26} =$	81.8	kPa
From Eq. 8:	$\gamma = 0.665\ 10^{-3}\ (81.8) =$	0.054	kPa $°C^{-1}$

The average value of the atmospheric pressure is 81.8 kPa.
The psychrometric constant, γ, is 0.054 kPa/°C.

Calculation procedures

Mean saturation vapour pressure (e_s)

As saturation vapour pressure is related to air temperature, it can be calculated from the air temperature. The relationship is expressed by:

$$e^{o}(T) = 0.6108 \exp\left[\frac{17.27\ T}{T + 237.3}\right] \qquad (11)$$

where	$e^{o}(T)$	saturation vapour pressure at the air temperature T [kPa],
	T	air temperature [°C],
	exp[..]	2.7183 (base of natural logarithm) raised to the power [..].

Values of saturation vapour pressure as a function of air temperature are given in Annex 2 (Table 2.3). Due to the non-linearity of the above equation, the mean saturation vapour pressure for a day, week, decade or month should be computed as the mean between the saturation vapour pressure at the mean daily maximum and minimum air temperatures for that period:

$$e_s = \frac{e^{o}(T_{max}) + e^{o}(T_{min})}{2} \qquad (12)$$

EXAMPLE 3
Determination of mean saturation vapour pressure

The daily maximum and minimum air temperature are respectively 24.5 and 15°C. Determine the saturation vapour pressure for that day.

From Eq. 11	$e°(T_{max}) = 0.6108 \exp[17.27(24.5)/(24.5+237.3)]$	3.075	kPa
From Eq. 11	$e°(T_{min}) = 0.6108 \exp[17.27(15)/(15+237.3)]$	1.705	kPa
From Eq. 12	$e_s = (3.075 + 1.705)/2$	2.39	kPa
	Note that for temperature 19.75°C (which is T_{mean}), e°(T) =	2.30	kPa

The mean saturation vapour pressure is 2.39 kPa.

Slope of saturation vapour pressure curve (Δ)

For the calculation of evapotranspiration, the slope of the relationship between saturation vapour pressure and temperature, Δ, is required. The slope of the curve (Figure 11) at a given temperature is given by.

$$\Delta = \frac{4098\left[0.6108\exp\left(\frac{17.27\ T}{T+237.3}\right)\right]}{(T+237.3)^2} \qquad (13)$$

where Δ slope of saturation vapour pressure curve at air temperature T [kPa $°C^{-1}$],
T air temperature [°C],
exp[..] 2.7183 (base of natural logarithm) raised to the power [..].

Values of slope Δ for different air temperatures are given in Annex 2 (Table 2.4). In the FAO Penman-Monteith equation, where Δ occurs in the numerator and denominator, the slope of the vapour pressure curve is calculated using mean air temperature (Equation 9).

Actual vapour pressure (e_a) derived from psychrometric data

The actual vapour pressure can be determined from the difference between the dry and wet bulb temperatures, the so-called wet bulb depression. The relationship is expressed by the following equation:

$$e_a = e^o(T_{wet}) - \gamma_{psy}(T_{dry} - T_{wet}) \quad (15)$$

where	e_a	actual vapour pressure [kPa],
	$e°(T_{wet})$	saturation vapour pressure at wet bulb temperature [kPa],
	γ_{psy}	psychrometric constant of the instrument [kPa °C^{-1}],
	T_{dry}-T_{wet}	wet bulb depression, with T_{dry} the dry bulb and T_{wet} the wet bulb temperature [°C].

The psychrometric constant of the instrument is given by:

$$\gamma_{psy} = a_{psy} P \quad (16)$$

e_a – pressão parcial de vapor, em KPa

$$e_a = (Ur_{med} \times e_s)/ 100$$

Actual vapour pressure (e_a) derived from relative humidity data

The actual vapour pressure can also be calculated from the relative humidity. Depending on the availability of the humidity data, different equations should be used.

• **For RH_{max} and RH_{min}:**

$$e_a = \frac{e^o(T_{min})\frac{RH_{max}}{100} + e^o(T_{max})\frac{RH_{min}}{100}}{2} \qquad (17)$$

where	e_a	actual vapour pressure [kPa],
	$e°(T_{min})$	saturation vapour pressure at daily minimum temperature [kPa],
	$e°(T_{max})$	saturation vapour pressure at daily maximum temperature [kPa],
	RH_{max}	maximum relative humidity [%],
	RH_{min}	minimum relative humidity [%].

For periods of a week, ten days or a month, RH_{max} and RH_{min} are obtained by dividing the sum of the daily values by the number of days in that period.

EXAMPLE 6
Determination of vapour pressure deficit

Determine the vapour pressure deficit with the data of the previous example (Example 5).

From Example 5:	$e°(T_{min})$ =	2.064	kPa
	$e°(T_{max})$ =	3.168	kPa
	e_a =	1.70	kPa
	$e_s - e_a = (2.064 + 3.168)/2 - 1.70$ =	0.91	kPa

The vapour pressure deficit is 0.91 kPa.

BOX 7
Calculation sheet for vapour pressure deficit (e_s - e_a)

Saturation vapour pressure: e_s (Eq. 11 or Table 2.3)

T_{max}		°C	$e^o(T_{max}) = 0.6108 \exp\left[\frac{17.27\ T_{max}}{T_{max} + 237.3}\right]$		kPa
T_{min}		°C	$e^o(T_{min}) = 0.6108 \exp\left[\frac{17.27\ T_{min}}{T_{min} + 237.3}\right]$		kPa
saturation vapour pressure $e_s = [e°(T_{max})+e°(T_{min})]/2$ Eq.12					kPa

TABLE 3
Conversion factors for radiation

	multiplier to obtain energy received on a unit surface per unit time				equivalent evaporation
	MJ m^{-2} day^{-1}	J cm^{-2} day^{-1}	cal cm^{-2} day^{-1}	W m^{-2}	mm day^{-1}
1 MJ m^{-2} day^{-1}	1	100	23.9	11.6	0.408
1 cal cm^{-2} day^{-1}	4.1868 10^{-2}	4.1868	1	0.485	0.0171
1 W m^{-2}	0.0864	8.64	2.06	1	0.035
1 mm day^{-1}	2.45	245	58.5	28.4	1

Calculation procedures

Extraterrestrial radiation for daily periods (R_a)

The extraterrestrial radiation, R_a, for each day of the year and for different latitudes can be estimated from the solar constant, the solar declination and the time of the year by:

$$R_a = \frac{24\,(60)}{\pi} G_{sc}\, d_r \left[\omega_s \sin(\varphi)\sin(\delta) + \cos(\varphi)\cos(\delta)\sin(\omega_s)\right] \qquad (21)$$

where

where	R_a	extraterrestrial radiation [MJ m^{-2} day^{-1}],
	G_{sc}	solar constant = 0.0820 MJ m^{-2} min^{-1},
	d_r	inverse relative distance Earth-Sun (Equation 23),
	ω_s	sunset hour angle (Equation 25 or 26) [rad],
	φ	latitude [rad] (Equation 22),
	δ	solar declination (Equation 24) [rad].

R_a is expressed in the above equation in MJ m^{-2} day^{-1}. The corresponding equivalent evaporation in mm day^{-1} is obtained by multiplying R_a by 0.408 (Equation 20). The latitude, φ, expressed in radians is positive for the northern hemisphere and negative for the southern hemisphere (Example 7). The conversion from decimal degrees to radians is given by:

$$[\text{Radians}] = \frac{\pi}{180}\,[\text{decimal degrees}] \qquad (22)$$

EXAMPLE 7
Conversion of latitude in degrees and minutes to radians

Express the latitudes of Bangkok (Thailand) at 13°44'N and Rio de Janeiro (Brazil) at 22°54'S in radians.

Latitude	Bangkok (northern hemisphere)	Rio de Janeiro (southern hemisphere)
degrees & minutes decimal degrees radians	13°44'N 13 + 44/60 = 13.73 (π/180) 13.73 = + 0.240	22°54'S (-22) + (-54/60) = - 22.90 (π/180) (-22.90) = - 0.400

The latitudes of Bangkok and Rio de Janeiro are respectively + 0.240 and - 0.400 radians.

The inverse relative distance Earth-Sun, d_r, and the solar declination, δ, are given by:

$$d_r = 1 + 0.033 \cos\left(\frac{2\,\pi}{365}\,J\right) \qquad (23)$$

$$\delta = 0.409 \sin\left(\frac{2\,\pi}{365}\,J - 1.39\right) \qquad (24)$$

where J is the number of the day in the year between 1 (1 January) and 365 or 366 (31 December). Values for J for all days of the year and an equation for estimating J are given in Annex 2 (Table 2.5).

The sunset hour angle, ω_s, is given by:

$$\omega_s = \arccos\left[-\tan(\varphi)\tan(\delta)\right] \qquad (25)$$

EXAMPLE 8
Determination of extraterrestrial radiation

Determine the extraterrestrial radiation (R_a) for 3 September at 20°S.

From Eq. 22 From Table 2.5:	20°S or φ = (π/180) (-20) = (the value is negative for the southern hemisphere) The number of day in the year, J =	- 0.35 246	rad days
From Eq. 23 From Eq. 24	d_r = 1 + 0.033 cos(2π(246)/365) = δ = 0.409 sin(2π(246)/365-1.39) =	0.985 0.120	rad rad
From Eq. 25: Then: and:	ω_s = arccos[-tan(-0.35)tan(0.120)] = sin(φ)sin(δ) = cos(φ)cos(δ) =	1.527 -0.041 0.933	rad - -
From Eq. 21	R_a = 24(60)/π (0.0820)(0.985)[1.527 (-0.041)+0.933sin(1.527)] =	32.2	MJ m^{-2} d^{-1}
From Eq. 20	expressed as equivalent evaporation = 0.408 (32.2)=	13.1	mm/day

The extraterrestrial radiation is 32.2 MJ m^{-2} day^{-1}.

Daylight hours (N)

The daylight hours, N, are given by:

$$N = \frac{24}{\pi} \omega_s \tag{34}$$

where ω_s is the sunset hour angle in radians given by Equation 25 or 26. Mean values for N (15th day of each month) for different latitudes are given in Annex 2, Table 2.7.

EXAMPLE 9
Determination of daylight hours

Determine the daylight hours (N) for 3 September at 20°S.

From Example 8:	ω_s = arccos[-tan(-0.35)tan(0.120)] =	1.527	rad
From Eq. 34:	N = 24/π (1.527) =	11.7	hour

The number of daylight hours is 11.7 hours.

Net solar or net shortwave radiation (R_{ns})

The net shortwave radiation resulting from the balance between incoming and reflected solar radiation is given by:

$$R_{ns} = (1 - \alpha) R_s \tag{38}$$

where R_{ns} net solar or shortwave radiation [MJ m^{-2} day^{-1}],

α albedo or canopy reflection coefficient, which is 0.23 for the hypothetical grass reference crop [dimensionless],

R_s the incoming solar radiation [MJ m^{-2} day^{-1}].

R_{ns} is expressed in the above equation in MJ m^{-2} day^{-1}.

Net longwave radiation (R_{nl})

The rate of longwave energy emission is proportional to the absolute temperature of the surface raised to the fourth power. This relation is expressed quantitatively by the Stefan-Boltzmann law. The net energy flux leaving the earth's surface is, however, less than that emitted and given by the Stefan-Boltzmann law due to the absorption and downward radiation from the sky. Water vapour, clouds, carbon dioxide and dust are absorbers and emitters of longwave radiation. Their concentrations should be known when assessing the net outgoing flux. As humidity and cloudiness play an important role, the Stefan-Boltzmann law is corrected by these two factors when estimating the net outgoing flux of longwave radiation. It is thereby assumed that the concentrations of the other absorbers are constant:

$$R_{nl} = \sigma\left[\frac{T_{max,K}{}^4 + T_{min,K}{}^4}{2}\right]\left(0.34 - 0.14\sqrt{e_a}\right)\left(1.35\frac{R_s}{R_{so}} - 0.35\right) \quad (39)$$

where

R_{nl}	net outgoing longwave radiation [MJ m^{-2} day^{-1}],
σ	Stefan-Boltzmann constant [4.903 10^{-9} MJ K^{-4} m^{-2} day^{-1}],
$T_{max,K}$	maximum absolute temperature during the 24-hour period [K = °C + 273.16],
$T_{min,K}$	minimum absolute temperature during the 24-hour period [K = °C + 273.16],
e_a	actual vapour pressure [kPa],
R_s/R_{so}	relative shortwave radiation (limited to ≤ 1.0),
R_s	measured or calculated (Equation 35) solar radiation [MJ m^{-2} day^{-1}],
R_{so}	calculated (Equation 36 or 37) clear-sky radiation [MJ m^{-2} day^{-1}].

EXAMPLE 11
Determination of net longwave radiation

In Rio de Janeiro (Brazil) at a latitude of 22°54'S (= -22.70°), 220 hours of bright sunshine, a mean monthly daily maximum and minimum air temperature of 25.1 and 19.1°C and a vapour pressure of 2.1 kPa were recorded in May. Determine the net longwave radiation.

From Example 10:	R_s =	14.5	MJ m^{-2} day^{-1}
From Eq. 36:	R_{so} = 0.75 R_a = 0.75 . 25.1 =	18.8	MJ m^{-2} day^{-1}
From Table 2.8 or for:	σ =	4.903 10^{-9}	MJ K^{-4} m^{-2} day^{-1}
Then:	T_{max} = 25.1°C =	298.3	K
and:	$\sigma T_{max,K}^4$ =	38.8	MJ m^{-2} day^{-1}
and:	T_{min} = 19.1°C =	292.3	K
and:	$\sigma T_{min,K}^4$ = 35.8 MJ m^{-2} day^{-1}	35.8	MJ m^{-2} day^{-1}
and:	e_a =	2.1	kPa
and:	0.34 - 0.14 $\sqrt{e_a}$ =	0.14	-
and:	R_s/R_{so} = (14.5)/(18.8)	0.77	-
-	1.35(0.77)-0.35 =	0.69	-
From Eq. 39:	R_{nl} = [(38.7 + 35.7)/2] (0.14) (0.69) =	3.5	MJ m^{-2} day^{-1}
From Eq. 20:	expressed as equivalent evaporation = 0.408 (3.5) =	1.4	mm/day

The net longwave radiation is 3.5 MJ m^{-2} day^{-1}.

Net radiation (R_n)

The net radiation (R_n) is the difference between the incoming net shortwave radiation (R_{ns}) and the outgoing net longwave radiation (R_{nl}):

$$R_n = R_{ns} - R_{nl} \qquad (40)$$

EXAMPLE 12 **Determination of net radiation** Determine the net radiation in Rio de Janeiro in May with the data from previous examples.			
From Example 10: From Eq. 39: From Example 11: From Eq. 40:	R_s = R_{ns} = (1 - 0.23) R_s = R_{nl} = R_n = 11.1 - 3.5 =	14.5 11.1 3.5 7.6	MJ m^{-2} day^{-1} MJ m^{-2} day^{-1} MJ m^{-2} day^{-1} MJ m^{-2} day^{-1}
From Eq. 20:	expressed as equivalent evaporation = 0.408 (7.7) =	3.1	mm/day
The net radiation is 7.6 MJ m^{-2} day^{-1}.			